

Body of Knowledge and Skills (BOKS)

voor de HBO-opleiding

Toegepaste wiskunde / Mathematical Engineering

*Bijlage bij Toegepaste wiskunde in perspectief, beroepsprofiel voor de HBO-opleiding Toegepaste wiskunde
(Mathematical Engineering)*

versie februari 2021

DE HAAGSE
HOGESCHOOL


Fontys


hogeschool



Hogeschool van Amsterdam


hogeschool

Inhoud

Inleiding

Kennisbasis SEFI

1 Algebra	Core level 0		
2 Analysis and calculus	Core level 0	Core level 1	Level 2
3 Discrete mathematics	Core level 0	Core level 1	Level 2
4 Geometry and trigonometry	Core level 0		
5 Linear algebra		Core level 1	Level 2
6 Statistics and Probability	Core Level 0	Core level 1	Level 2

Kennisbasis Focusgebieden Toegepaste Wiskunde

- 7 Financiële Wiskunde
- 8 Data Science en Statistiek
- 9 Operations Research en Logistiek
- 10 ICT
- 11 Engineering Mathematics

Vaardigheden

- 12 Wiskundige vaardigheden
- 13 Soft skills

INLEIDING

Een werkgroep van SEFI (Europese organisatie voor Engineering opleidingen) heeft in 2013 een nieuwe versie uitgebracht van 'A Framework for Mathematics Curricula in Engineering Education'. In hoofdstuk 3 van dit SEFI-framework is een kennisbasis beschreven op de terreinen Algebra, Analyse en Calculus, Discrete wiskunde, Meetkunde en goniometrie, Lineaire algebra, Kansrekening en statistiek.

De inhoud van deze wiskundige terreinen is in het SEFI-framework verdeeld in vier niveaus: 0, 1, 2 en 3. De opvolging in niveaus representeert de moeilijkheidsgraad. Voor niveaus 0, 1, en 2 zijn de leeruitkomsten gedetailleerd beschreven. De relevante onderdelen van deze niveaus zijn opgenomen in de kennisbasis van Toegepaste Wiskunde. Voor de terreinen Algebra, Analyse en Calculus, Discrete wiskunde, Meetkunde en goniometrie, Lineaire algebra, Kansrekening en Statistiek zijn de onderdelen in de BOKS dus een deelverzameling van de onderdelen van het SEFI-framework. Dit deel van de BOKS vormt de basis van alle vijf de Toegepaste Wiskunde opleidingen.

Het hoogste niveau (3) staat voor realistische beroepscontexten voor een afgestudeerde ('integration with real-life engineering examples'). Dit niveau 3 is in het SEFI-framework alleen globaal beschreven met de gedachte dat opleidingen dat zelf nader invullen. Toegepaste Wiskunde heeft hier invulling aangegeven door in de BOKS leeruitkomsten te beschrijven voor vijf beroepscontexten, de zogenaamde *focusgebieden*. De keuze voor deze focusgebieden is ingegeven door de overweging dat in deze focusgebieden een relatief grote vraag naar toegepast wiskundigen bestaat. Met het oog op recente ontwikkelingen in het werkveld zijn in 2020 de focusgebieden herzien. De actuele focusgebieden zijn: Financiële Wiskunde, Data Science en Statistiek, Operations Research en Logistiek, ICT, en Engineering Mathematics. Een uitgebreide beschrijving van de focusgebieden kan worden gevonden in 'Toegepaste Wiskunde in perspectief: beroepsprofiel voor de HBO-opleiding Toegepaste Wiskunde (Mathematical Engineering)' van het Landelijk Overleg Toegepaste Wiskunde (2020). Per hogeschool waar de opleiding wordt aangeboden kan de nadruk op de diverse focusgebieden verschillen. De opleidingen kunnen zich profileren door zich te specialiseren in één of meerdere focusgebieden. Afhankelijk van het accent van de opleiding, zal elke opleiding een andere selectie aanbieden van de beschreven leeruitkomsten.

Naast de kennis-onderdelen 'wiskundekennis algemeen' en 'focusgebieden', zijn ook vaardigheden beschreven, zowel algemene wiskundige vaardigheden als soft skills. Deze vaardigheden vormen een integraal onderdeel van alle Toegepaste Wiskunde opleidingen. De opgenomen wiskundige vaardigheden (hoofdstuk 12) zijn ontleend aan hoofdstuk 2 (General Mathematical Competencies for Engineers) uit 'A Framework for Mathematics Curricula in Engineering Education'. De lijst met soft skills (hoofdstuk 13) is gedeeltelijk gebaseerd op de inhoud van de volgende boeken:

- Rien Elling, Bas Andeweg, 2012, Report Writing for Readers with Little Time.
- Roel Grit, 2011, Projectmanagement
- Ben Baarda, 2014, Research: This Is It!

KENNISBASIS SEFI

Voor elk punt geldt voorafgaand de zin 'As a result of learning this material you should be able to'.

1 ALGEBRA

Core Level 0

1.1 Arithmetic of real numbers

- carry out the operations add, subtract, multiply and divide on both positive and negative numbers
- express an integer as a product of prime factors
- calculate the highest common factor and lowest common multiple of a set of integers
- obtain the modulus of a number
- understand the rules governing the existence of powers of a number
- combine powers of a number
- evaluate negative powers of a number
- express a fraction in its lowest form
- carry out arithmetic operations on fractions
- represent roots as fractional powers
- express a fraction in decimal form and vice-versa
- carry out arithmetic operations on numbers in decimal form
- round numerical values to a specified number of decimal places or significant figures
- understand the concept of ratio and solve problems requiring the use of ratios
- understand the scientific notation form of a number
- manipulate logarithms
- understand how to estimate errors in measurements and how to combine them.

1.2 Algebraic expressions and formulae

- add and subtract algebraic expressions and simplify the result
- multiply two algebraic expressions, removing brackets
- evaluate algebraic expressions using the rules of precedence
- change the subject of a formula
- distinguish between an identity and an equation
- obtain the solution of a linear equation
- recognise the kinds of solution for two simultaneous equations
- understand the terms direct proportion and inverse proportion
- solve simple problems involving proportion
- factorise a quadratic expression
- carry out the operations add, subtract, multiply and divide on algebraic fractions
- interpret simple inequalities in terms of intervals on the real line
- solve simple inequalities, both geometrically and algebraically
- interpret inequalities which involve the absolute value of a quantity.

1.3 Linear laws

- understand the Cartesian co-ordinate system
- plot points on a graph using Cartesian co-ordinates
- understand the terms 'gradiënt' and 'intercept' with reference to straight lines
- obtain and use the equation $y = mx + c$
- obtain and use the equation of a line with known gradient through a given point
- obtain and use the equation of a line through two given points
- use the intercept form of the equation of a straight line
- use the general equation $ax + by + c = 0$

- determine algebraically whether two points lie on the same side of a straight line
- recognise when two lines are parallel
- recognise when two lines are perpendicular
- obtain the solution of two simultaneous equations in two unknowns using graphical and algebraic methods
- interpret simultaneous linear inequalities in terms of regions in the plane
- reduce a relationship to linear form.

1.4 Quadratics, cubics, polynomials

- recognise the graphs of $y = x^2$ and of $y = -x^2$
- understand the effects of translation and scaling on the graph of $y = x^2$
- rewrite a quadratic expression by completing the square
- use the rewritten form to sketch the graph of the general expression of $ax^2 + bx + c$
- determine the intercepts on the axes of the graph of $y = ax^2 + bx + c$
- determine the highest or lowest point on the graph of $y = ax^2 + bx + c$
- sketch the graph of a quadratic expression
- state the criterion that determines the number of roots of a quadratic equation
- solve the equation of $ax^2 + bx + c = 0$ via factorisation, by completing the square and by the formula
- recognise the graphs of $y = x^3$ and of $y = -x^3$
- recognise the main features of the graph of $y = ax^3 + bx^2 + cx + d$
- state and use the remainder theorem
- factorise simple polynomials as a product of linear and quadratic factors.

2 Analysis and Calculus

Core Level 0

2.1 Functions and their inverses

- define a function, its domain and its range
- use the notation $f(x)$
- determine the domain and range of simple functions
- relate a pictorial representation of a function to its graph and to its algebraic definition
- determine whether a function is injective, surjective, bijective
- understand how a graphical translation can alter a functional description
- understand how a reflection in either axis can alter a functional description
- understand how a scaling transformation can alter a functional description
- determine the domain and range of simple composite functions
- use appropriate software to plot the graph of a function
- obtain the inverse of a function by a pictorial representation, graphically or algebraically
- determine the domain and range of the inverse of a function
- determine any restrictions on $f(x)$ for the inverse to be a function
- obtain the inverse of a composite function
- recognise the properties of the function $1/x$
- understand the concept of the limit of a function.

2.2 Sequences, series, binomial expansions

- define a sequence and a series and distinguish between them
- recognise an arithmetic progression and its component parts
- find the general term of an arithmetic progression
- find the sum of an arithmetic series
- recognise a geometric progression and its component parts

- find the general term of a geometric progression
- find the sum of a finite geometric series
- interpret the term 'sum' in relation to a infinite geometric series
- find the sum of an infinite geometric series when it exists
- find the arithmetic mean of two numbers
- find the geometric mean of two numbers

2.3 Logarithmic and exponential functions

- recognise the graphs of the power law function
- define the exponential function and sketch its graph
- define the logarithmic function as the inverse of the exponential function
- use the laws of logarithms to simplify expressions
- solve equations involving exponential and logarithmic functions
- solve problems using growth and decay models.

2.4 Rates of change and differentiation

- define average and instantaneous rates of change of a function
- understand how the derivative of a function at a point is defined
- recognise the derivative of a function as the instantaneous rate of change
- interpret the derivative as the gradient at a point on a graph
- distinguish between 'derivative' and 'derived function'
- use the notations $\frac{dy}{dx}$, $f'(x)$, $y'(x)$ etc.
- use a table of the derived functions of simple functions
- recall the derived function of each of the standard functions
- use the multiple, sum, product and quotient rules
- use the chain rule
- relate the derivative of a function to the gradient of a tangent to its graph
- obtain the equation of the tangent and normal to the graph of a function.

2.5 Stationary points, maximum and minimum values

- use the derived function to find where a function is increasing or decreasing
- define a stationary point of a function
- distinguish between a turning point and a stationary point
- locate a turning point using the first derivative of a function
- classify turning points using first derivatives
- obtain the second derived function of simple functions
- classify stationary points using second derivatives.

2.6 Indefinite integration

- reverse the process of differentiation to obtain an indefinite integral for simple functions
- understand the role of the arbitrary constant
- use a table of indefinite integrals of simple functions
- understand and use the notation for indefinite integrals
- use the constant multiple rule and the sum rule
- use indefinite integration to solve practical problems such as obtaining velocity from a formula for acceleration or displacement from a formula for velocity.

2.7 Definite integration, applications to areas and volumes

- understand the idea of a definite integral as the limit of a sum
- realise the importance of the Fundamental Theorem of the Calculus
- obtain definite integrals of simple functions
- use the main properties of definite integrals

- calculate the area under a graph and recognise the meaning of a negative value
- calculate the area between two curves
- calculate the volume of a solid of revolution
- use trapezium and Simpson's rules to approximate the value of an indefinite integral

2.8 Complex numbers

- define a complex number and identify its component parts
- represent a complex number on an Argand diagram
- carry out the operations of addition and subtraction
- write down the conjugate of a complex number and represent it graphically
- identify the modulus and argument of a complex number
- carry out the operations of multiplication and division in both Cartesian and polar form
- solve equations of the form $z^n = a$, where a is a real number.

2.9 Proof

- understand how a theorem is deduced from a set of assumptions
- appreciate how a corollary is developed from a theorem
- follow a proof of Pythagoras' theorem
- follow proofs of theorems for example, the concurrency of lines related to triangles and/or the equality of angles related to circles.

Core Level 1

2.10 Rational functions

- sketch the graph of a rational function where the numerator is a linear expression and the denominator is either a linear expression or the product of two linear expressions
- obtain the partial fractions of a rational function, including cases where the denominator has a repeated linear factor or an irreducible quadratic factor.

2.11 Complex numbers

- state and use Euler's formula
- state and use De Moivre's theorem for a rational index
- find the roots of a complex number

2.12 Functions

- define and recognise an odd function and an even function
- understand the properties 'convex' and 'concave'
- identify, from its graph, where a function is concave and where it is convex
- define and locate points of inflection on the graph of a function
- evaluate a function of two or more variables at a given point
- relate the main features, including stationary points, of a function of 2 variables to its 3D plot and to a contour map
- obtain the first partial derivatives of simple functions of several variables

2.13 Differentiation

- understand the concepts of continuity and smoothness
- differentiate functions defined implicitly
- locate any points of inflection of a function

2.14 Sequences and series

- understand convergence and divergence of a sequence
- know what is meant by a partial sum

- understand the concept of a power series
- apply simple tests for convergence of a series
- find the tangent and quadratic approximations to a function
- understand the idea of radius of convergence of a power series
- recognise Maclaurin series for standard functions
- understand how Maclaurin series generalise to Taylor series
- use Taylor series to obtain approximate percentage changes in a function.

2.15 Methods of integration

- obtain definite and indefinite integrals of rational functions in partial fraction form
- apply the method of integration by parts to indefinite and definite integrals
- use the method of substitution on indefinite and definite integrals
- solve practical problems which require the evaluation of an integral
- recognise simple examples of improper integrals
- use the formula for the maximum error in a trapezoidal rule estimate
- use the formula for the maximum error in a Simpson's rule estimate.

2.16 Applications of integration

- find the length of part of a plane curve
- find the curved surface area of a solid of revolution

Level 2

2.17 Ordinary differential equations

- understand how rates of change can be modelled using first and second derivatives
- recognise the kinds of boundary condition which apply in particular situations
- distinguish between boundary and initial conditions
- distinguish between general solution and particular solution
- understand how existence and uniqueness relate to a solution
- classify differential equations and recognise the nature of their general solution
- understand how substitution methods can be used to simplify ordinary differential equations

2.18 First order ordinary differential equations

- recognise when an equation can be solved by separating its variables
- obtain general solutions of equations by applying the method
- obtain particular solutions by applying initial conditions
- recognise the common equations of the main areas of application
- interpret the solution and its constituent parts in terms of the physical problem
- understand the term 'exact equation'
- obtain the general solution to an exact equation
- solve linear differential equations using integrating factors
- find and interpret solutions to equations describing standard physical situations

2.19 Linear optimisation

- recognise a linear programming problem in words and formulate it mathematically
- represent the feasible region graphically
- solve a maximisation problem graphically by superimposing lines of equal profit
- carry out a simple sensitivity analysis
- represent and solve graphically a minimisation problem
- explain the term redundant constraint
- understand the meaning and use of slack variables in reformulating a problem
- understand the concept of duality and be able to formulate the dual to a given problem.

2.20 The simplex method

- convert a linear programming problem into a simplex tableau
- solve a maximisation problem by the simplex method
- interpret the tableau at each stage of the journey round the simplex
- recognise cases of failure
- write down the dual to a linear programming problem
- use the dual problem to solve a minimisation problem.

3 Discrete Mathematics

Core Level 0

3.1 Sets

- understand the concepts of a set, a subset and the empty set
- determine whether an item belongs to a given set or not
- use and interpret Venn diagrams
- find the union and intersection of two given sets
- apply the laws of set algebra.

Core Level 1

3.2 Mathematical logic

- recognise a proposition
- negate a proposition
- form a compound proposition using the connectives AND, OR, IMPLICATION
- construct a truth table for a compound proposition
- construct a truth table for an implication
- verify the equivalence of two propositions using a truth table
- identify a contradiction and a tautology
- construct the converse of a proposition
- obtain the contrapositive form of an implication
- understand the universal quantifier 'for all'
- understand the existential quantifier 'there exists'
- negate propositions with quantifiers
- follow simple examples of direct and indirect proof
- follow a simple example of a proof by contradiction.

3.3 Mathematical induction and recursion

- understand (weak) mathematical induction
- follow a simple proof which uses mathematical induction
- define a set by induction
- use structural induction to prove some simple properties of a set which is given by induction.
- understand the concept of recursion
- define the factorial of a positive integer by recursion (any other suitable example will serve just as well).

3.4 Graphs

- recognise a graph (directed and/or undirected) in a real situation
- understand the notions of a path and a cycle
- understand the notion of a tree and a binary tree

Level 2

3.5 Number systems

- carry out arithmetic operations in the binary system
- carry out arithmetic operations in the hexadecimal system
- use Euclid's algorithm for finding the greatest common divisor

3.6 Graphs

- recognise an Euler trail in a graph and / or an Euler graph
- recognise a Hamilton cycle (path) in a graph
- find components of connectivity in a graph
- find a minimal spanning tree of a given connected graph.

3.7 Algorithms

- understand when an algorithm solves a problem
- understand the 'big O' notation for functions
- understand the worst case analysis of an algorithm
- understand one of the sorting algorithms
- understand the idea of depth-first search
- understand the idea of breadth-first search
- understand a multi-stage algorithm (for example, finding the shortest path, finding the minimal spanning tree or finding maximal flow)
- understand the notion of a polynomial-time-solvable problem
- understand the notion of an NP problem (as a problem for which it is 'easy' to verify an affirmative answer)
- understand the notion of an NP-complete problem (as a hardest problem among NP problems).

4 Geometry and Trigonometry

Core Level 0

4.1 Trigonometry

- define the sine, cosine and tangent of an acute angle
- define the reciprocal ratios cosecant, secant and cotangent
- state and use the fundamental identities arising from Pythagoras' theorem
- relate the trigonometric ratios of an angle to those of its complement
- relate the trigonometric ratios of an angle to those of its supplement
- state in which quadrants each trigonometric ratio is positive (the CAST rule)
- state and apply the sine rule
- state and apply the cosine rule
- calculate the area of a triangle from the lengths of two sides and the included angle
- solve a triangle given sufficient information about its sides and angles
- recognise when there is no triangle possible and when two triangles can be found.

4.2 Trigonometric functions and applications

- define the term *periodic function*
- sketch the graphs of $\sin(x)$, $\cos(x)$ and $\tan(x)$ and describe their main features
- deduce the graphs of the reciprocal functions cosec, sec and cot
- deduce the nature of the graphs of $a \cdot \sin(x)$, $a \cdot \cos(x)$, $a \cdot \tan(x)$
- deduce the nature of the graphs of $\sin(ax)$, $\cos(ax)$, $\tan(ax)$
- deduce the nature of the graphs of $\sin(x + a)$, $a + \sin(x)$, etc.
- solve the equations $\sin(x) = c$, $\cos(x) = c$, $\tan(x) = c$

- use the expression $a \cdot \sin(\omega t + \varphi)$ to represent an oscillation and relate the parameters to the motion
- rewrite the expression $a \cdot \cos \omega t + b \cdot \sin(\omega t)$ as a single cosine or sine formula.

5 Linear Algebra

Core Level 1

5.1 Vector arithmetic

- distinguish between vector and scalar quantities
- understand and use vector notation
- represent a vector pictorially
- carry out addition and scalar multiplication and represent them pictorially
- determine the unit vector in a specified direction
- represent a vector in component form (two and three components only).

5.2 Vector algebra and applications

- solve simple problems in geometry using vectors
- solve simple problems using the component form (for example, in mechanics)
- define the scalar product of two vectors and use it in simple applications
- understand the geometric interpretation of the scalar product
- define the vector product of two vectors and use it in simple applications
- understand the geometric interpretation of the vector product
- define the scalar triple product of three vectors and use it in simple applications
- understand the geometric interpretation of the scalar triple product.

5.3 Matrices and determinants

- understand what is meant by a matrix
- recall the basic terms associated with matrices (for example, diagonal, trace, square, triangular, identity)
- obtain the transpose of a matrix
- determine any scalar multiple of a matrix
- recognise when two matrices can be added and find, where possible, their sum
- recognise when two matrices can be multiplied and find, where possible, their product
- calculate the determinant of 2×2 and 3×3 matrices
- understand the geometric interpretation of 2×2 and 3×3 determinants
- use the elementary properties of determinants in their evaluation
- state the criterion for a square matrix to have an inverse
- write down the inverse of a 2×2 matrix when it exists
- determine the inverse of a matrix, when it exists, using row operations
- calculate the rank of a matrix
- use appropriate software to determine inverse matrices.

5.4 Solution of simultaneous linear equations

- represent a system of linear equations in matrix form
- understand how the general solution of an inhomogeneous linear system of m equations in n unknowns is obtained from the solution of the homogeneous system and a particular solution
- recognise the different possibilities for the solution of a system of linear equations
- give a geometrical interpretation of the solution of a system of linear equations
- understand how and why the rank of the coefficient matrix and the augmented matrix of a linear system can be used to analyse its solution
- use the inverse matrix to find the solution of 3 simultaneous linear equations when possible
- understand the term 'ill-conditioned'

- apply the Gauss elimination method and recognise when it fails
- use appropriate software to solve simultaneous linear equations.

5.5 Least squares curve fitting

- define the least squares criterion for fitting a straight line to a set of data points
- understand how and why the criterion is satisfied by the solution of $A^T Ax = A^T b$
- understand the effect of outliers
- modify the method to deal with polynomial models
- use appropriate software to fit a straight line to a set of data points.

Level 2

5.6 Eigenvalue problems

- interpret eigenvectors and eigenvalues of a matrix in terms of the transformation it represents
- convert a transformation into a matrix eigenvalue problem
- find the eigenvalues and eigenvectors of 2x2 and 3x3 matrices algebraically
- use appropriate software to compute the eigenvalues and eigenvectors of a matrix

6 Statistics and Probability

Core Level 0

6.1 Data Handling

- interpret data presented in the form of line diagrams, bar charts, pie charts
- interpret data presented in the form of stem and leaf diagrams, box plots, histograms
- construct line diagrams, bar charts, pie charts, stem and leaf diagrams, box plots, histograms for suitable data sets
- calculate the mode, median and mean for a set of data items.

6.2 Probability

- define the terms 'outcome', event and probability.
- calculate the probability of an event by counting outcomes
- calculate the probability of the complement of an event
- calculate the probability of the union of two mutually-exclusive events
- calculate the probability of the union of two events
- calculate the probability of the intersection of two independent events.

Core Level 1

6.3 Data Handling

- calculate the range, inter-quartile range, variance and standard deviation for a set of data items
- distinguish between a population and a sample
- know the difference between the characteristic values (moments) of a population and of a sample
- construct a suitable frequency distribution from a data set
- calculate relative frequencies
- calculate measures of average and dispersion for a grouped set of data
- understand the effect of grouping on these measures.

6.4 Combinatorics

- evaluate the number of ways of arranging unlike objects in a line
- evaluate the number of ways of arranging objects in a line, where some are alike
- evaluate the number of ways of arranging unlike objects in a ring
- evaluate the number of ways of permuting r objects from n unlike objects
- evaluate the number of combinations of r objects from n unlike objects
- use the multiplication principle for combinations.

6.5 Simple probability

- interpret probability as a degree of belief
- understand the distinction between *a priori* and *a posteriori* probabilities
- use a tree diagram to calculate probabilities
- know what conditional probability is and be able to use it (Bayes' theorem)
- calculate probabilities for series and parallel connections.

6.6 Probability models

- define a random variable and a discrete probability distribution
- state the criteria for a binomial model and define its parameters
- calculate probabilities for a binomial model
- state the criteria for a Poisson model and define its parameters
- calculate probabilities for a Poisson model
- state the expected value and variance for each of these models
- understand when a random variable is continuous
- explain the way in which probability calculations are carried out in the continuous case.

6.7 Normal distribution

- handle probability statements involving continuous random variables
- convert a problem involving a normal variable to the area under part of its density curve
- relate the general normal distribution to the standardised normal distribution
- use tables for the standardised normal variable
- solve problems involving a normal variable using tables.

6.8 Sampling

- define a random sample
- know what a sampling distribution is
- understand the term 'mean squared error' of an estimate
- understand the term 'unbiasedness' of an estimate

6.9 Statistical inference

- apply confidence intervals to sample estimates
- follow the main steps in a test of hypothesis.
- understand the difference between a test of hypothesis and a significance test (pvalue)
- define the level of a test (error of the first kind)
- define the power of a test (error of the second kind)
- state the link between the distribution of a normal variable and that of means of samples
- place confidence intervals around the sample estimate of a population mean
- test claims about the population mean using results from sampling
- recognise whether an alternative hypothesis leads to a one-tail or a two-tail test
- compare the approaches of using confidence intervals and hypothesis tests.

Level 2

6.10 One-dimensional random variables

- compare empirical and theoretical distributions
- apply the exponential distribution to simple problems
- apply the normal distribution to simple problems
- apply the Weibull distribution to simple problems
- apply the gamma distribution to simple problems.

6.11 Two-dimensional random variables

- understand the concept of a joint distribution
- understand the terms 'joint density function', 'marginal distribution functions'
- define independence of two random variables
- solve problems involving linear combinations of random variables
- determine the covariance of two random variables
- determine the correlation of two random variables.

6.12 Small sample statistics

- realise that the normal distribution is not reliable when used with small samples
- use tables of the t -distribution
- solve problems involving small-sample means using the t -distribution
- use tables of the F -distribution
- use pooling of variances where appropriate
- use the method of pairing where appropriate.

6.13 Small sample statistics: chi-square tests

- use tables for chi-squared distributions
- decide on the number of degrees of freedom appropriate to a particular problem
- use the chi-square distribution in tests of independence (contingency tables)
- use the chi-square distribution in tests of goodness of fit.

6.14 Analysis of variance

- set up the information for a one-way analysis of variance
- interpret the ANOVA table
- solve a problem using one-way analysis of variance
- set up the information for a two-way analysis of variance
- solve a problem using two-way analysis of variance.

6.15 Simple linear regression

- derive the equation of the line of best fit to a set of data pairs
- calculate the correlation coefficient
- place confidence intervals around the estimates of slope and intercept
- place confidence intervals around values estimated from the regression line
- carry out an analysis of variance to test goodness of fit of the regression line
- interpret the results of the tests in terms of the original data
- describe the relationship between linear regression and least squares fitting.

6.16 Multiple linear regression and design of experiments

- understand the ideas involved in a multiple regression analysis
- appreciate the importance of experimental design
- recognise simple statistical designs.

KENNISBASIS FOCUSGEBIEDEN TOEGEPASTE WISKUNDE

7 Financiële Wiskunde

Actuariële theorie

- Actuariële berekeningen kunnen maken met behulp van sterfte- en overlevingskansen op basis van een periodetafel en een prognosetafel.
- Het waarderen van geldstromen en het omgaan met begrippen als oprenten en disconteren, zowel met een vaste rente als met een rentetermijnstructuur.

Levensverzekeringen

- De kenmerken (her)kennen van verschillende levensverzekeringen, waaronder lijfrente, overlijdensrisicoverzekering, nabestaandenpensioen, en erfrente.
- Het kunnen bepalen van de Netto en Bruto Koopsom voor deze verzekeringen.
- Het kunnen bepalen van de Netto en Bruto Premie voor deze verzekeringen.
- Het kunnen berekenen van de voorziening.
- Het kunnen rekenen aan mutaties op deze verzekeringen, b.v. premievrij maken.

Schadeverzekeringen

- Algemene kennis over schadeverzekeringen (o.a. omtrent producten, verzekeringstechnische aspecten, premie, schade en resultaatbepaling).
- Het kunnen bepalen en interpreteren van de voorziening en IBN(E)R m.b.v. Chain Ladder methode.
- Het kunnen bepalen van de risicopremie voor schadeverzekeringen m.b.v. de bijbehorende schade methodieken.
- De kenmerken, de werking en de voor- en nadelen van de verschillende herverzekeringsvormen met elkaar te vergelijken.
- Het kunnen berekenen en interpreteren van de Solvency Capital Requirement (SCR) m.b.v. de gedelegeerde verordeningen van de Solvency II wet- en regelgeving.

Softwaretoepassingen

- Het implementeren van een softwaretool (bijvoorbeeld in Excel) waarin berekeningen aan levensverzekeringen kunnen worden geautomatiseerd.

Beleggingstheorie

- De kenmerken (her)kennen van verschillende beleggingscategoriën, waaronder aandelen, opties, en obligaties.
- Het kunnen waarderen van deze beleggingen, waaronder Black-Scholes.
- Eenvoudige berekeningen kunnen uitvoeren in het kader van Balansmanagement, bijvoorbeeld duration matching.
- Kennen van de basis van Portefeuilletheorie (Markowitz, Index Model, CAPM) en de hierin geldende wiskundige mechanismen en begrippen, waaronder:
 - Het principe van risico diversificatie in een portfolio (Markowitz, Index model).
 - Grafisch bepalen van de ligging van de optimale portfolio in een verwachte opbrengst vs standaard deviatie grafiek en hierbij de relatie tot maximeren van de Sharpe Ratio kunnen uitleggen (Markowitz).
 - De verwachte opbrengst en standaard deviatie van een gegeven portfolio berekenen

- rekening houdend met covarianties tussen beleggingscategorieën.
- Een regressiemodel met verwachte return als functie van de bèta van een beleggingsproduct en de verwachte opbrengst van de markt kunnen interpreteren in relatie tot bedrijfsspecifiek en marktgerelateerd risico (Index Model).
- De uitgangspunten kennen die worden verondersteld in CAPM.
- De reward to risk ratio van een markt kunnen berekenen en interpreteren (CAPM).

8 Data Science and Statistiek

Data engineering

- Inrichten van geschikte infrastructuur voor opslag en analyse van grote hoeveelheden data.
- Ontwerpen van een datawarehouse passend bij een informatiebehoefte.
- Verzamelen en opslaan van (streaming) data door middel van ETL technieken.
- Selecteren en toepassen van datavoorbewerkingstechnieken, waaronder outlier identificatie en missende waarde analyse, om data geschikt te maken voor analyse.
- Toepassen van software voor data analyse.

Statistiek

- Analyseren van een data set door middel van beschrijvende statistiek.
- Effectief visualiseren van data door middel van grafieken.
- Betrouwbaarheidsintervallen berekenen voor gemiddeldes, proporties, en standaard deviaties.
- Relaties tussen variabele identificeren door middel van correlatie, factor analyse, en principale-componenten analyse.
- Gebruik van parametrische en non-parametrische hypothesetoetsen om de significantie van resultaten te bepalen voor één, twee, of meerdere samples bestaande uit numerieke of categoriale data:
 - Selecteren van een geschikte hypothesetoets voor een probleem en data set.
 - Bepalen of een verdeling voldoet aan de aannames een hypothesetoets.
 - Handmatig toepassen van een hypothesetoets op een kleine data set.
 - Toepassen van een hypothesetoets op een data set door middel van statistische software.
 - Interpreteren van de resultaten van een hypothesetoets in de context van het originele probleem.
- Opstellen van lineaire, multivariate, en logistische regressiemodellen door middel van statistische software.
- Verifiëren van de aannames van regressiemodellen.
- Berekenen van betrouwbaarheidsintervallen en de accuraatheid van regressiemodellen.
- Interpreteren van de resultaten van regressiemodellen.
- Opstellen van een tijdreeksmodel met een trend- en een seizoenscomponent.
- Bepalen van de nauwkeurigheid van een tijdreeksmodel.

Business Intelligence en Machine Learning

- Vertalen van een bedrijfsprobleem naar een Data Science opdracht.
- Business Intelligence technieken toepassen voor beslissingsondersteuning in een bedrijf.
- Ontwerpen en realiseren van een dashboard passend bij een informatiebehoefte.
- Selecteren en toepassen van unsupervised data mining technieken om patronen in data te vinden, waaronder:
 - K-means clusteren;

- Hiërarchische clusteren;
- Association rule mining;
- Selecteren en toepassen van supervised data mining technieken om een voorspellend model op te stellen, waaronder:
 - Beslisbomen en random forest;
 - Neurale netwerken / deep learning;
 - Nearest neighbor technieken;
 - SVM.
- Evalueren van de resultaten van een getraind model in de context van het originele bedrijfsprobleem.

9 Operations Research en Logistiek

Lineaire, geheeltallige en binaire programmeringsmodellen

- Bij een gegeven context kunnen opstellen van lineaire, geheeltallige en binaire programmeringsmodellen;
- Binaire variabelen kunnen gebruiken voor modelleringen zoals als-dan, keuze uit alternatieven, etc.
- Voor niet al te grote problemen oplossings technieken voor lineaire, geheeltallige en binaire programmeringsproblemen kunnen begrijpen en uitvoeren zoals grafische methode, simplex-methode en de branch-and-bound-methode.

Wachtrijmodellen

- Bij een gegeven context kunnen opstellen van een wachtrijmodel en kennen van de Kendall-Lee-notatie;
- Voor een wachtrijmodel bepalen van geschikte kansverdelingen, zoals de negatief exponentiële, en het schatten van de parameters daarvan;
- Voor relatief eenvoudige wachtrijmodellen kunnen uitvoeren van analytische berekeningen aan wachttijd, wachtrijlengte, etc.
- De evenwichtskansen bepalen en de stelling van Little toepassen bij wachtrijsystemen met negatief-exponentiële verdelingen voor tussenaankomsttijden en bedieningstijden bij één of meerdere bedienden;
- Het begrijpen van en kunnen rekenen aan Markovketens.

Andere modellen en technieken

- Het kunnen opstellen van een simulatiemodel en het methodisch correct uitvoeren en interpreteren van simulatie-experimenten;
- Het kunnen opstellen van dynamische programmeringsmodellen, zowel deterministisch als probabilistisch;
- Het kunnen toepassen van metaheuristieken zoals Tabu Search, Simulated Annealing en Genetisch Algoritme;
- Het kunnen opstellen en analyseren van beslissingsbomen;
- Het analyseren van netwerken voor projectmanagement zoals de kritieke pad methode en de PERT aanpak.

Algemeen

- De genoemde modellen en technieken in diverse logistieke contexten kunnen gebruiken, zoals bij plannings-, voorraad- en distributievraagstukken;

- Kunnen bepalen welk type model(len) in een bepaalde context geschikt kunnen zijn;
- Het kunnen gebruiken van solvers om problemen op te lossen;
- De oplossingen kunnen vertalen naar de praktijk.

10 ICT

Databases

- Een database ontwerpen voor een bedrijfsprobleem door middel van technieken voor datamodellering.
- Het gebruik van SQL voor het bevragen en beheren van databases.

Software ontwikkeling

- Vertalen van een bedrijfsprobleem naar requirements voor een software applicatie.
- Een gegeven algoritme implementeren in een programmeertaal gebruikmakend van programmeertechnieken, zoals loops, functies, en variabelen.
- Ontwerpen en implementeren van een applicatie gebruikmakend van gestandaardiseerde programmeerstructuren en design patterns.
- Selecteren en toepassen van datastructuren en algoritmes in software ontwikkeling.
- Implementeren van ontwikkelde wiskundige modellen en algoritmes in een applicatie.

Informatietheorie

- Kennis van verschillende getallenstelsels en informatiecoderingstechnieken.
- Berekenen van de complexiteit en schaalbaarheid van algoritmes.

11 Engineering Mathematics

Scientific Computing

- Efficiënt en gebruikersvriendelijk implementeren van numerieke methoden voor partiële differentiaalvergelijking met directe en indirecte solvers en geschikt gekozen software.

Signaal- en systeemanalyse en regeltechniek

- Modelleren en analyseren van elementaire discrete en continue signalen in tijddomein en frequentiedomein.
- Modelleren en analyseren van lineair tijdinvariante digitale en analoge systemen met behulp van begrippen als convolutie, impulsresponsie, stapresponsie, frequentieresponsie, overdrachtsfunctie, Laplace- en Fouriertransformaties en Kalmanfilters.
- Ontwikkelen van regelaars voor systemen door analyse van deze systemen in tijddomein en frequentiedomein.

Vectoranalyse

- Dubbelintegralen, triple-integralen, lijnintegralen en oppervlakte-integralen kunnen opstellen en berekenen in Cartesische, pool-, cilinder-, en bolcoördinaten voor vectorvelden en scalarvelden om grootheden als oppervlakte, volume, massa, arbeid, booglengte en flux te berekenen.
- Krommen en oppervlakken parametriseren in 2 en 3 dimensies teneinde deze te kunnen visualiseren.

Partiële differentiaalvergelijkingen

- Een wiskundig model met behulp van partiële differentiaalvergelijkingen opstellen voor fysische systemen uit disciplines als (robot)kinematica, stromingsleer en elektriciteit.
- Modellen met partiële differentiaalvergelijkingen numeriek oplossen met behulp van eindige differentiemethoden en eindige elementen methoden met eigen geschreven code en/of specifieke softwaretools.
- Resultaten van numerieke berekeningen postprocessen en inschatten op realiteit.

VAARDIGHEDEN

12 Wiskundige vaardigheden

12.1 Thinking mathematically

This competency comprises a knowledge of the kind of questions that are dealt with in mathematics and the types of answers mathematics can and cannot provide, and the ability to pose such questions. It includes the recognition of mathematical concepts and an understanding of their scope and limitations as well as extending the scope by abstraction and generalisation of results. This also includes an understanding of the certainty mathematical considerations can provide.

12.2 Reasoning mathematically

This competency includes on the one hand the ability to understand and assess an already existing mathematical argumentation (chain of logical arguments), in particular to understand the notion of proof and to recognise the central ideas in proofs. It also includes the knowledge and ability to distinguish between different kinds of mathematical statements (definition, if-then-statement, iff-statement etc.). On the other hand it includes the construction of chains of logical arguments and hence of transforming heuristic reasoning into own proofs (reasoning logically).

12.3 Posing and solving mathematical problems

This competency comprises on the one hand the ability to identify and specify mathematical problems (be they pure or applied, open-ended or closed) and on the other hand the ability to solve mathematical problems (including knowledge of the adequate algorithms). What really constitutes a problem is not well defined and it depends on personal capabilities whether or not a question is considered as a problem. This has to be borne in mind, for example when identifying problems for a certain group of students.

12.4 Modelling mathematically

This competency also has essentially two components: the ability to analyse and work in existing models (find properties, investigate range and validity, relate to modelled reality) and the ability to 'perform active modelling' (structure the part of reality that is of interest, set up a mathematical model and transform the questions of interest into mathematical questions, answer the questions mathematically, interpret the results in reality and investigate the validity of the model, monitor and control the whole modelling process).

12.5 Representing mathematical entities

This competency includes the ability to understand and use mathematical representations (be they symbolic, numeric, graphical and visual, verbal, material objects etc.) and to know their relations, advantages and limitations. It also includes the ability to choose and switch between representations based on this knowledge.

12.6 Handling mathematical symbols and formalism

This competency includes the ability to understand symbolic and formal mathematical language and its relation to natural language as well as the translation between both. It also includes the rules of formal mathematical systems and the ability to use and manipulate symbolic statements and expressions according to the rules.

12.7 Communicating in, with, and about mathematics

This competency includes on the one hand the ability to understand mathematical statements (oral, written or other) made by others and on the other hand the ability to express oneself mathematically in different ways.

12.8 Making use of aids and tools

This competency includes knowledge about the aids and tools that are available as well as their potential and limitations. Additionally, it includes the ability to use them thoughtfully and efficiently.

13 Soft skills

13.1 Onderzoek

- De verschillende onderdelen van de onderzoekscyclus herkennen.
- Literatuur zoeken en gebruiken.
- Onderzoeksvragen opstellen op basis van een bedrijfsprobleem.
- Verschillende methodes om een stelling te onderbouwen herkennen en toepassen.
- Geschikte onderzoeksmethodes kiezen en toepassen.
- Alternatieven afwegen.
- Conclusies trekken uit resultaten.
- Aanbevelingen doen op basis van resultaten.
- Zelfstandig nieuwe kennis en vaardigheden leren.
- Verbinden van theorie en praktijk.

13.2 Project management

- Een projectplan schrijven.
- Verschillende rollen binnen een project benoemen.
- Een planning en taakverdeling maken en gedurende een project bijhouden.
- Vergaderen.
- Effectief omgaan met conflicten.
- Ethische en maatschappelijk consequenties van een project benoemen en betrekken in een beslissingsproces.
- Bij verschil van inzicht tot een compromis komen.

13.3 Communicatie

- Correct Nederlands of Engels schrijven.
- Correct Nederlands of Engels spreken.
- Een gestructureerd rapport schrijven.
- Een presentatie houden.
- Een stelling beargumenteren.
- Solliciteren.
- Advies uitbrengen aan verschillende doelgroepen.
- Reflecteren op het eigen leerproces.